Standard Method of Test for

Determining Dynamic Modulus of Hot Mix Asphalt (HMA)

AASHTO Designation: T 342-11 (2015)

1. SCOPE

1.1. This test method covers procedures for preparing and testing hot mix asphalt (HMA) to determine the dynamic modulus and phase angle over a range of temperatures and loading frequencies.

1.2. This standard is applicable to laboratory-prepared specimens of mixtures with nominal maximum size aggregate less than or equal to 37.5 mm (1.48 in.).

1.3. *This standard may involve hazardous material, operations, and equipment. This standard does not purport to address all of the safety concerns associated with its use. It is the responsibility of the user of this procedure to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. REFERENCED DOCUMENTS

2.1. *AASHTO Standards*:

- R 30, Mixture Conditioning of Hot Mix Asphalt (HMA)
- T 166, Bulk Specific Gravity (G_{mb}) of Compacted Hot Mix Asphalt (HMA) Using Saturated Surface-Dry Specimens
- T 209, Theoretical Maximum Specific Gravity (G_{mm}) and Density of Hot Mix Asphalt (HMA)
- T 269, Percent Air Voids in Compacted Dense and Open Asphalt Mixtures
- T 312, Preparing and Determining the Density of Asphalt Mixture Specimens by Means of the Superpave Gyratory Compactor

2.2. *ASTM Standard*:

- E4, Standard Practices for Force Verification of Testing Machines

2.3. *Other Document*:

- Chapra, Steven C. and Raymond P. Canale, *Numerical Methods for Engineers,* The McGraw-Hill Companies, Inc., New York, NY, 1985, pp. 404–407.

3. TERMINOLOGY

3.1. *Definitions*:

3.1.1. *complex modulus* (E^*)—a complex number that defines the relationship between stress and strain for a linear viscoelastic material.

3.1.2. *dynamic modulus* (|E^*|)—the normal value of the complex modulus calculated by dividing the maximum (peak-to-peak) stress by the recoverable (peak-to-peak) axial strain for a material subjected to a sinusoidal loading.

3.1.3. *phase angle* (ϕ)—the angle in degrees between a sinusoidal applied peak stress and the resulting peak strain in a controlled stress test.

4. SUMMARY OF METHOD

4.1. A sinusoidal (haversine) axial compressive stress is applied to a specimen of asphalt concrete at a given temperature and loading frequency. The applied stress and the resulting recoverable axial strain response of the specimen is measured and used to calculate the dynamic modulus and phase angle.

4.2. Figure 1 presents one schematic of the dynamic modulus test.

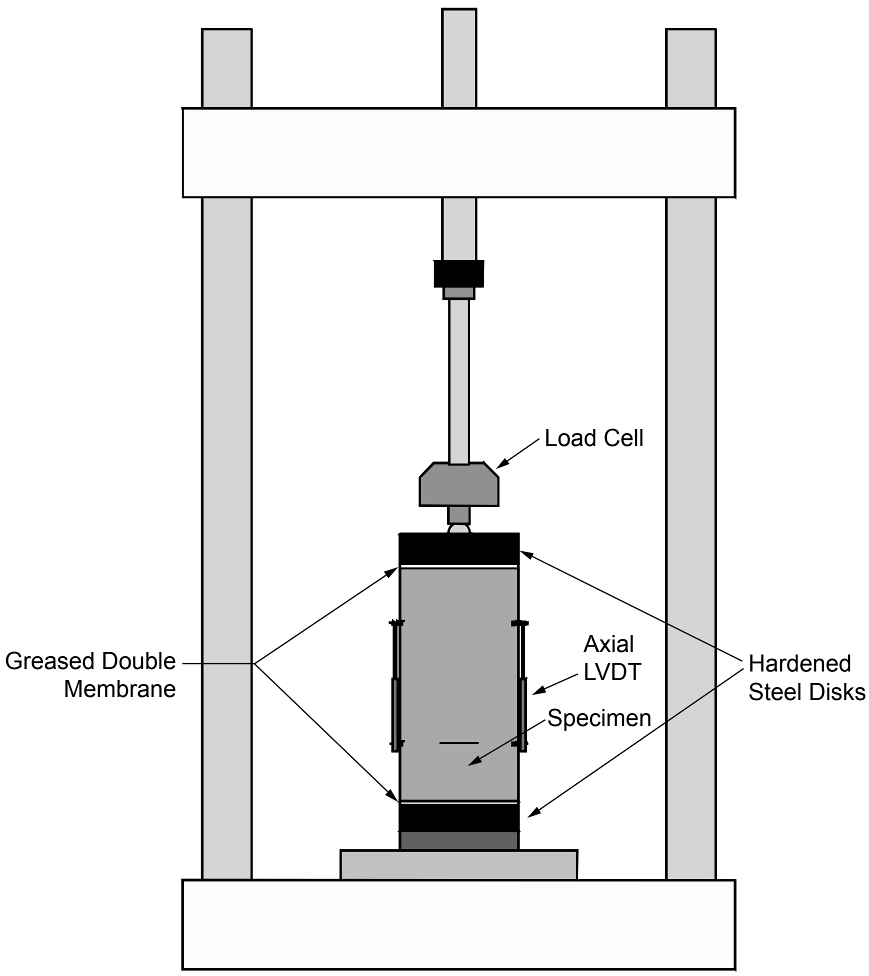

Figure 1—General Schematic of Dynamic Modulus Test

5. SIGNIFICANCE AND USE

5.1. Dynamic modulus values measured over a range of temperatures and frequencies of loading can be shifted into a master curve for characterizing asphalt concrete for pavement thickness design and performance analysis.

5.2. The values of dynamic modulus and phase angle can also be used as performance criteria for HMA design.

6. APPARATUS

6.1. *Dynamic Modulus Test System*—A dynamic modulus test system consisting of a testing machine, environmental chamber, and measuring system.

6.2. *Testing Machine*—A servohydraulic testing machine capable of producing a controlled haversine compressive loading. The testing machine should have a capability of applying load over a range of frequencies from 0.1 to 25 Hz and stress level up to 2800 kPa (400 psi). For sinusoidal loads, the standard error of the applied load shall be less than 5 percent. The standard error of the applied load is a measure of the difference between the measured load data and the best-fit sinusoid. The standard error of the load is defined in Equation 1.

$$se(P) = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}{n-4}}\left(\frac{100\%}{\hat{x}_o}\right) \quad (1)$$

where:

$se(P)$ = standard error of the applied load,

x_i = measured load at point i,

$\hat{x}_i$ = predicted load at point i from the best-fit sinusoid,

n = total number of data points collected during test, and

$\hat{x}_o$ = amplitude of the best-fit sinusoid.

6.2.1. *Environmental Chamber*—A chamber for controlling the test specimen at the desired temperature. The environmental chamber shall be capable of controlling the temperature of the specimen over a temperature range from –10 to 60°C (14 to 140°F) to an accuracy of ±0.5°C (±1°F). The chamber shall be large enough to accommodate the test specimen and a dummy specimen with thermocouple mounted at the center for temperature verification.

6.2.2. *Measurement System*—The system shall be fully computer-controlled, capable of measuring and recording the time history of the applied load and the axial deformations. The system shall be capable of measuring the period of the applied sinusoidal load and resulting deformations with a resolution of 0.5 percent. The accuracy and resolution of measurements are summarized in Table 1.

Table 1—Accuracy and Resolution of Measurement System

Measurement	Range	Accuracy	Resolution
Load	0.12 to 25 kN	Error ≤0.0 percent	≤0.0012 kN
Deformation	≥1 mm	Error ≤0.0025 mm	≤0.0002 mm
Inherent phase lag between load and deformation	Not specified	≤1 degree	Not specified

6.2.2.1. *Load*—The load shall be measured with an electronic load cell in contact with one of the specimen caps. The load cell shall be calibrated in accordance with ASTM E4. The load measuring system shall have a minimum range of 0 to 25 kN (0 to 5600 lb) with a resolution of 1.2 N (0.24 lb).

6.2.2.2. *Axial Deformations*—Axial deformations shall be measured with linear variable differential transformers (LVDT) mounted between gauge points glued to the specimen, for example, as shown in Figure 2.

The deformations shall be measured at two locations 180 degrees apart, three locations 120 degrees apart, or four locations 90 degrees apart. The measurement setup that calls for four locations set at 90 degrees apart has an advantage over the other two options in that, in case one LVDT does not function properly, LVDT and the LVDT on the opposite side can be dropped, and the remaining two LVDTs can be used to determine the average deformation. The LVDTs shall have a range of ±0.5 mm (±0.02 in.). The deformation measuring system shall have auto zero and selectable ranges as defined in Table 2.

Table 2—Deformation Measuring System Requirements

Range, mm (in.)	Resolution, mm (in.)
±0.5 (0.01969)	0.0100 (0.00039)
±0.25 (0.00984)	0.0050 (0.00020)
±0.125 (0.00492)	0.0025 (0.00010)
±0.0625 (0.00246)	0.0010 (0.00004)

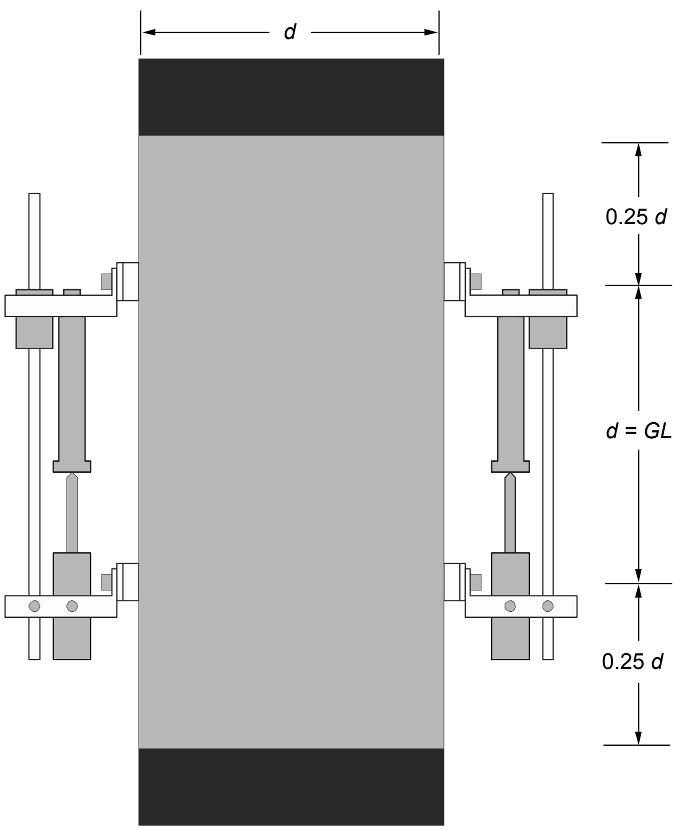

Figure 2—General Schematic of Gauge Points (not to scale)

6.2.3. *Loading Platens*—Loading platens, sized 104.5 ± 0.5 mm, are required above and below the specimen to transfer the load from the testing machine to the specimen. Generally, these platens should be made of hardened or plated steel, or anodized high-strength aluminum. Softer materials will require more frequent replacement. Materials that have linear elastic modulus properties and hardness properties lower than that of 6061-T6 aluminum shall not be used.

6.2.4. *End Treatment*—Friction-reducing end treatments shall be placed between the specimen ends and the loading platens. The end treatments shall consist of two TFE-fluorocarbon sheets or two 0.5-mm (0.02-in.) thick latex membranes separated with silicone grease.

6.3. *Superpave Gyratory Compactor*—A gyratory compactor and associated equipment for preparing laboratory specimens in accordance with T 312. The compactor shall be capable of compacting 170-mm (6.7-in.) high specimen.

6.4. *Saw*—A machine for sawing test specimen ends to the appropriate length is required. The saw shall have a diamond cutting edge and shall be capable of cutting specimens to the prescribed dimensions without excessive heating or shock.

Note 1—A diamond masonry saw greatly facilitates the preparation of test specimens with smooth, parallel ends. Both single- or double-bladed diamond saws should have feed mechanisms and speed controls of sufficient precision to ensure compliance with Sections 9.5 and 9.6 of this method. Adequate blade stiffness is also important to control flexing of the blade during thin cuts.

6.5. *Core Drill*—A coring machine with cooling system and a diamond bit for cutting nominal 101.6-mm (4.00-in.) diameter test specimens.

Note 2—A coring machine with adjustable vertical feed and rotational speed is recommended. The variable feeds and speeds may be controlled by various methods. A vertical feed rate of approximately 0.05 mm/rev (0.002 in./rev) and a rotational speed of approximately 450 r/min has been found to be satisfactory for several Superpave mixtures. Use of a standard electric core drill with a holder for the specimen is also acceptable.

7. HAZARDS

7.1. Observe standard laboratory safety precautions when preparing and testing hot-mix asphalt (HMA) specimens.

8. TESTING EQUIPMENT CALIBRATION

8.1. The signal conditioning and data acquisition device of the testing system shall be checked to ensure that there is no excess phase shift between load and displacement channels.

8.2. The testing system shall be calibrated prior to initial use and at least once a year thereafter or per manufacturer requirements or per every 200 tests.

8.3. Verify the capability of the environmental chamber to maintain the required temperature within the accuracy specified.

8.4. Verify the calibration of all measurement components (such as load cell and specimen deformation measurement device) of the testing system.

8.5. If any of the verifications yield data that do not comply with the accuracy specified, correct the problem prior to proceeding with testing.

9. TEST SPECIMENS

9.1. *Size*—Dynamic modulus testing shall be performed on test specimens cored from gyratory 150-mm (6-in.) compacted mixtures. The average diameter of the test specimens shall be between 100 and 104 mm (3.94 and 4.1 in.) with a standard deviation of 1.0 mm (0.04 in.). The average height of the test specimen shall be between 147.5 and 152.5 mm (5.81 and 6.00 in.).

9.2. *Aging*—Laboratory-prepared mixtures shall be temperature-conditioned in accordance with the 4-h short-term oven conditioning procedure in R 30. Field mixtures need not be aged prior to testing.

9.3. *Gyratory Specimens*—Prepare 170-mm (6.7-in.) tall specimens to the required air void content in accordance with T 312.

Note 3—Testing should be performed on test specimens (101.6-mm (4.0-in.) diameter) meeting specific air void tolerances. The gyratory specimen (152.4-mm (6.0-in.) diameter) air void content required to obtain a specified test specimen air void content must be determined by trial and error, achieved by using less or more mixture and compacted to the same height in the gyratory compactor. Generally, the test specimen air void content is 1.5 to 2.5 percent lower than the air void content of the gyratory specimen when the test specimen is removed from the middle as specified in this test method.

9.4. *Coring*—Core the nominal 101.6-mm (4.0-in.) diameter test specimens from the center of the gyratory specimens. Both the core drill and the gyratory specimen should be adequately supported to ensure that the resulting test specimen is cylindrical with sides that are smooth, parallel, and free from steps, ridges, and grooves.

9.5. *Diameter*—Measure the diameter of the test specimen at the mid-height and third points along axes that are 90 degrees apart. Record each of the six measurements to the nearest 1 mm (0.04 in.). Calculate the average and the standard deviation of the six measurements. If the standard deviation is greater than 2.5 mm (0.10 in.), discard the specimen. For acceptable specimens, the average diameter reported to the nearest 1 mm (0.04 in.) shall be used in all material property calculations.

9.6. *End Preparation*—The ends of all test specimens shall be smooth and perpendicular to the axis of the specimen. Prepare the ends of the specimen by sawing with a single- or double-bladed saw. The prepared specimen ends shall meet the tolerances described below. Reject test specimens not meeting these tolerances.

9.6.1. The specimen ends shall have a cut surface waviness height within a tolerance of ±0.05 mm (±0.002 in.) across any diameter. This requirement shall be checked in a minimum of three positions at approximately 120-degree intervals using a straightedge and feeler gauges approximately 8.1 to 12.5 mm (0.32 to 0.49 in.) wide or an optical comparator.

9.6.2. The specimen end shall not depart from perpendicular to the axis of the specimen by more than 1 degree, equivalent to 2.7 mm in 152.4 mm (0.11 in. in 6.10 in.). This requirement shall be checked on each specimen using a machinist's square and feeler gauges.

9.7. *Air Void Content*—Determine the air void content of the final test specimen in accordance with T 269. Reject specimens with air voids that differ by more than 0.5 percent from the target air voids.

Note 4—Considerable time can be saved if the cored test specimens were treated as wet, and the weights in water and saturated surface dry were measured immediately or within a short time period after coring. The test specimens can then be left to dry overnight, the dry weight can be measured the next day, and then they can be immediately prepared for testing.

9.8. *Replicates*—The number of test specimens required depends on the number of axial strain measurements made per specimen and the desired accuracy of the average dynamic modulus. Three replicate specimens should be tested to obtain a desired accuracy limit (e.g., less than ±15 percent of the true dynamic modulus). Table 3 summarizes the estimated accuracy associated with the number of specimens.

Table 3—Estimated Accuracy Related to the Number of Specimens

LVDTs per Specimen	Number of Specimens	Estimated Limit of Accuracy, %
2	2	±18.0
2	3	±15.0
2	4	±13.4
3	2	±13.1
3	3	±12.0
3	4	±11.5

9.9. *Sample Storage*—If test specimens will not be tested within 2 days, wrap specimens in polyethylene and store in an environmentally protected storage area at temperatures between 5 and 15°C (40 and 60°F). Specimens shall not be stacked during storage.

Note 5—To eliminate effects of aging on test results, it is recommended that specimens be stored no more than 2 weeks prior to testing.

10. TEST SPECIMEN INSTRUMENTATION

10.1. Attach mounting studs for the axial LVDTs to the sides of the specimen with epoxy cement. Figure 3 shows details of the mounting studs and LVDT mounting hardware.

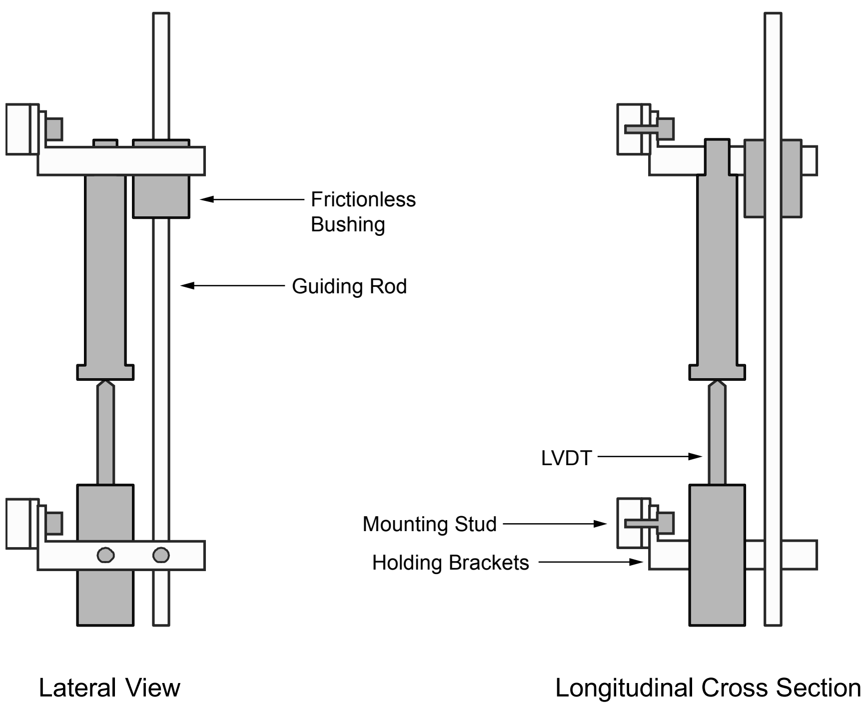

Figure 3—Mounting Hardware Details

Note 6—Quick-setting epoxy, such as Duro Master Mend Extra Strength Quick Set QM-50, has been found satisfactory for attaching studs.

10.2. The gauge length for measuring axial deformations shall be 101.6 mm ± 1 mm (4.00 in. ± 0.04 in.). Suitable alignment and spacing fixture shall be used to facilitate mounting of the axial deformation measuring hardware. The gauge length is measured between the stud centers.

11. PROCEDURE

11.1. The test series for the development of master curves for use in pavement response and performance analysis shall be conducted at –10, 4.4, 21.1, 37.8, and 54°C (14, 40, 70, 100, and 130°F) at loading frequencies of 0.1, 0.5, 1.0, 5, 10, and 25 Hz at each temperature. Each test specimen, individually instrumented with LVDT brackets, should be tested for each of the 30 combinations of temperature and frequency of loading starting with the lowest temperature and proceeding to the highest. Testing at a given temperature should begin with the highest frequency of loading and proceed to the lowest.

11.2. Place the test specimen in the environmental chamber and allow it to equilibrate to the specified testing temperature ±0.3°C (±1°F). A monitoring specimen with a thermocouple mounted at the center shall be used to determine when the specimen reaches the specified test temperature. Minimum recommended equilibrium temperature times are provided as a guideline. Note that these guidelines for equilibrium times are recommended when testing two to four replicates at a time.

Table 4—Recommended Equilibrium Times

Specimen Temperature, °C (°F)	Time from Room Temperature, h 25°C (77°F)	Time from Previous Test Temperature, h
–10 (14)	Overnight	Overnight
4 (40)	Overnight	4 h or overnight
21 (70)	1	3
37 (100)	2	2
54 (130)	3	1

11.3. Place one of the friction-reducing end treatments on top of the hardened steel disk at the bottom of the loading frame. Place the specimen on top of the lower end treatment, and mount the axial LVDTs to the hardware previously attached to the specimen. Adjust the LVDT to near the end of its linear range to allow the full range to be available for the accumulation of compressive permanent deformation.

11.4. Place the upper friction-reducing end treatment and hardened steel disk on top of the specimen. Center the specimen with the hydraulic load actuator visually in order to avoid eccentric loading.

11.5. Apply a contact load (P_{min}) equal to 5 percent of the dynamic load that will be applied to the specimen. It is acceptable to increase the applied contact stress to 20 kPa to improve machine control effectiveness by applying a load that will maintain positive contact with the specimen but will not damage the specimen.

11.6. Adjust and balance the electronic measuring system as necessary.

11.7. Apply sinusoidal (haversine) loading ($P_{dynamic}$) to the specimen in a cyclic manner. The dynamic load should be adjusted to obtain axial strains between 50 and 150 microstrain.

Note 7—The dynamic load depends upon the specimen stiffness and generally ranges between 15 and 2800 kPa (2 and 400 psi). Higher load is needed at colder temperatures. Table 5 presents typical dynamic stress levels based on temperature.

Table 5—Typical Dynamic Stress Levels

Temperature, °C (°F)	Range, kPa	Range, psi
–10 (14)	1400 to 2800	200 to 400
4 (40)	700 to 1400	100 to 200
21 (70)	350 to 700	50 to 100
37 (100)	140 to 250	20 to 50
54 (130)	35 to 70	5 to 10

11.8. Test the specimens from lowest to highest temperature, that is, from –10°C (14°F) to 54°C (130°F). At each temperature, apply the loading from highest to lowest frequency, that is, from 25 Hz to 0.1 Hz. At the beginning of testing, precondition the specimen with 200 cycles at 25 Hz at stress level corresponding to Table 5. Then load the specimen as specified in Table 6. A typical rest period between each frequency run is 2 min. This rest period shall not exceed 30 min for any two frequency runs.

Table 6—Number of Cycles for the Test Sequence

Frequency, Hz	Number of Cycles
25	200
10	200
5	100
1	20
0.5	15
0.1	15

11.9. The specimen shall be discarded at the end of any testing series at each temperature period. If the cumulative unrecovered permanent strain was found to be greater than 1500 micro units of strain, reduce the maximum loading stress level to half. Keep the test data up to this last resting period, discard the specimen, and use a new specimen for the rest of testing periods under reduced load conditions.

12. CALCULATIONS

12.1. This section presents a standard procedure for calculating both the dynamic modulus, $|E^*(\omega)|$, and the phase angle, $\theta(\omega)$, using data from a specific loading frequency, ω. It also defines four measures of data quality that should be used with the limits in Section 13 to evaluate the reliability of test data.

12.2. The general approach used here involves the least squares fit of a sinusoid, as described by Chapra and Canale in *Numerical Methods for Engineers* (McGraw–Hill, 1985, pp. 404–407). Regression is used because it is easy for most engineers and technicians in the paving industry to understand and apply effectively. This approach is easily performed on a spreadsheet.

12.3. The data produced from each dynamic modulus test at frequency ω_0 are stored in the form of several arrays, one for time $[t_i]$, one for stress $[\sigma_i]$, and one for each of the $j = 1, 2, 3,\ldots m$ strain transducers used $[\in_j]$. The number of $i = 1, 2, 3\ldots n$ points in each array will be equal and will depend upon the number of data points collected per loading cycle and on the total number of cycles for which data has been collected. It is recommended that 50 points per cycle and 5 cycles be used for a total of 250 data points.

12.4. *Analyze Stress Data*. The first step in the analysis is to analyze the data in the stress array. The data analysis is performed on centered stress data that are computed from the raw stress data by subtracting the average stress.

12.4.1. Determine the average stress as:

$$\bar{\sigma} = \frac{\sum_{i=1}^{n} \sigma_i}{n} \tag{2}$$

where:

$\bar{\sigma}$ = average stress;

σ_i = raw stress point i in the data array; and

n = number of points in the data array.

12.4.2. Then compute the centered stresses by subtracting the average stress from each of the stress measurements:

$$\sigma_i' = \sigma_i - \bar{\sigma} \tag{3}$$

where:

σ_i' = centered stress at point i in the data array;

σ_i = raw stress point i in the data array; and

$\bar{\sigma}$ = average stress.

12.4.3. From the centered stress data, compute three stress coefficients: offset, in-phase magnitude, and out-of-phase magnitude.

$$A_{\sigma 0} = \frac{\sum_{i=1}^{n} \sigma_i'}{n} \tag{4}$$

$$A_{\sigma 1} = \frac{2}{n} \sum_{i=1}^{n} \sigma_i' \cos\left(\omega_0 t_i\right) \tag{5}$$

$$B_{\sigma 1} = \frac{2}{n} \sum_{i=1}^{n} \sigma_i' \sin\left(\omega_0 t_i\right) \tag{6}$$

where:

$A_{\sigma 0}$ = stress offset coefficient, kPa (psi);

σ_i' = centered stress at point i in the data array;

$A_{\sigma 1}$ = stress in-phase magnitude coefficient, kPa (psi);

ω_0 = frequency of applied stress, rad/s;

t_i = time at point i in the data array, s; and

$B_{\sigma 1}$ = stress out-of-phase magnitude coefficient, kPa (psi).

12.4.4. From the stress coefficients, compute the stress magnitude and the stress phase angle.

$$\left|\sigma^*\right| = \sqrt{A_{\sigma 1}^2 + B_{\sigma 1}^2} \tag{7}$$

$$\theta_\sigma = \arctan\left(-\frac{B_{\sigma 1}}{A_{\sigma 1}}\right) \tag{8}$$

where:

$|\sigma^*|$ = stress magnitude, kPa (psi);

$A_{\sigma 1}$ = stress in-phase magnitude coefficient, kPa (psi);

$B_{\sigma 1}$ = stress out-of-phase magnitude coefficient, kPa (psi); and

θ_σ = stress phase angle, degrees.

12.4.5. Compute an array of predicted centered stresses and the standard error of the applied stress.

$$\hat{\sigma}_i' = A_{\sigma 0} + A_{\sigma 1}\cos\left(\omega_0 t_i\right) + B_{\sigma 1}\sin\left(\omega_0 t_i\right) \tag{9}$$

$$se\,(\sigma) = \sqrt{\frac{\sum_{i=1}^{n}(\hat{\sigma}_i' - \sigma_i')^2}{n-4}}\left(\frac{100\%}{\left|\sigma^*\right|}\right) \tag{10}$$

where:

$\hat{\sigma}_i'$ = predicted centered stress at point i, kPa (psi);

$A_{\sigma 0}$ = stress offset coefficient, kPa (psi);

$A_{\sigma 1}$ = stress in-phase magnitude coefficient, kPa (psi);

ω_0 = frequency of applied stress, rad/s;

t_i = time at point i in the data array, s;

$B_{\sigma 1}$ = stress out-of-phase magnitude coefficient, kPa (psi);

$se(\sigma)$ = standard error for the applied stress, percent;

σ_i' = centered stress at point i in the data array;

n = number of points in data array; and

$|\sigma^*|$ = stress magnitude, kPa (psi).

12.5. *Analyze Strain Data*. The second step in the analysis is to perform a similar analysis on the data from each of the strain transducers. However, in this case the data are corrected for drift caused by permanent deformation during the test, and centered data based on the average strain for the transducer.

12.5.1. To estimate the drift in the strain data, search each strain transducer array and determine local maximum and minimum values and the time when they occur for each loading cycle. Then determine the slope of the local maximum and minimum values with respect to time using linear regression. The average of these two slopes is the rate of drift D_j for strain transducer j.

12.5.2. Determine the average strain for each strain transducer as:

$$\overline{\epsilon}_j = \frac{\sum_{i=1}^{n} \epsilon_{j\,i}}{n} \tag{11}$$

where:

$\overline{\epsilon}_j$ = average strain for transducer j;

$\in_{j_i}$ = raw strain for transducer j at point i in data array; and

n = number of points in the data array.

12.5.3. Correct and center the strain data for each transducer by subtracting from the measured strains the rate of drift times the loading time and also subtracting the average strain for that transducer:

$$\in_{j_i}' = \in_{j_i} - D_j t_i - \overline{\in_j} \tag{12}$$

where:

$\in_{j_i}'$ = corrected and centered strain for transducer j at point i in data array;

$\in_{j_i}$ = raw strain for transducer j at point i in data array;

D_j = rate of drift for transducer j;

t_i = time for point i in data array; and

$\overline{\in_j}$ = average strain for transducer j.

12.5.4. From the corrected and centered strain data for each strain transducer, compute three strain coefficients: offset, in-phase magnitude, and out-of-phase magnitude.

$$A_{\in j0} = \frac{\sum_{i=1}^{n} \in_{j_i}'}{n} \tag{13}$$

$$A_{\in j1} = \frac{2}{n} \sum_{i=1}^{n} \in_{j_i}' \cos\left(\omega_0 t_i\right) \tag{14}$$

$$B_{\in j1} = \frac{2}{n} \sum_{i=1}^{n} \in_{j_i}' \sin\left(\omega_0 t_i\right) \tag{15}$$

where:

$A_{\in j0}$ = offset coefficient for strain transducer j;

$\in_{j_i}'$ = corrected and centered strain for transducer j at point i in data array;

$A_{\in j1}$ = in-phase magnitude coefficient for strain transducer j;

ω_0 = frequency of applied stress, rad/s;

t_i = time for point i in data array, s; and

$B_{\in j1}$ = out-of-phase magnitude coefficient for strain transducer j.

12.5.5. From the strain coefficients, compute the strain magnitude and the strain phase angle for each transducer.

$$\left|\in_j{}^*\right| = \sqrt{A_{\in j1}^2 + B_{\in j1}^2} \tag{16}$$

$$\theta_{\in j1} = \arctan\left(-\frac{B_{\in j1}}{A_{\in j1}}\right) \tag{17}$$

where:

$|\in_j{}^*|$ = strain magnitude for strain transducer j;

$A_{\in j1}$ = in-phase magnitude coefficient for strain transducer j;

$B_{\in_{j1}}$ = out-of-phase magnitude coefficient for strain transducer j; and

$\theta_{\in_j}$ = phase angle for strain transducer j, degrees.

12.5.6. For each strain transducer, compute an array of predicted corrected and centered strains and the standard error of the strain data.

$$\hat{\in}_{j_i}{}' = A_{\in_{j0}} + A_{\in_{j1}} \cos\left(\omega_0 t_i\right) + B_{\in_{j1}} \sin\left(\omega_0 t_i\right) \tag{18}$$

$$se(\in_j) = \sqrt{\frac{\sum_{i=1}^{n}\left(\hat{\in}_{j_i}{}' - \in_{j_i}{}'\right)^2}{n-4}}\left(\frac{100\%}{\left|\in_j *\right|}\right) \tag{19}$$

where:

$\hat{\in}_{j_i}{}'$ = predicted corrected and centered strain for strain transducer j at point i;

$A_{\in_{j0}}$ = offset coefficient for strain transducer j;

$A_{\in_{j1}}$ = in-phase magnitude coefficient for strain transducer j;

ω_0 = frequency of applied stress, rad/s;

t_i = time for point i in data array, s;

$B_{\in_{j1}}$ = out-of-phase magnitude coefficient for strain transducer j;

$se(\in_j)$ = standard error for strain transducer j response, percent;

$\in_{j_i}{}'$ = corrected and centered strain for transducer j at point i in data array;

n = number of points in data array; and

$|\in_j *|$ = strain magnitude for strain transducer j.

12.5.7. Calculate the average phase angle, strain magnitude, and standard error for all m strain transducers, along with two uniformity coefficients representing the variation among the strain transducers:

$$\overline{\theta_{\in}} = \frac{\sum_{j=1}^{m} \theta_{\in_j}}{m} \tag{20}$$

$$\overline{\left|\in *\right|} = \frac{\sum_{j=1}^{m}\left|\in_j *\right|}{m} \tag{21}$$

$$se\left(\in\right) = \frac{\sum_{j=1}^{m} se\left(\in_j\right)}{m} \tag{22}$$

$$U_{\in} = \sqrt{\frac{\sum_{j=1}^{m}\left(\left|\in_j *\right| - \overline{\left|\in *\right|}\right)^2}{m-1}}\left(\frac{100\%}{\overline{\left|\in *\right|}}\right) \tag{23}$$

$$U_\theta = \sqrt{\frac{\sum_{j=1}^{m}(\theta_{\in_j} - \overline{\theta_\in})^2}{m-1}} \quad (24)$$

where:

$\overline{\theta_\in}$ = average phase angle for all strain transducers, degrees;

m = number of strain transducers;

$\overline{|\in^*|}$ = average strain magnitude;

$se(\in)$ = average standard error for all strain transducers, percent;

$U_\in$ = uniformity coefficient for strain transducers, percent; and

U_θ = uniformity coefficient for phase angle, degrees.

12.6. *Compute Phase Angle and Dynamic Modulus*. The final step in the data analysis is to calculate the overall phase angle, $\theta(\omega)$, in degrees, and the complex modulus, $|E^*(\omega)|$, kPa (psi), at the selected frequency, ω:

$$\theta(\omega) = \overline{\theta_\in} - \theta_\sigma \quad (25)$$

$$|E^*(\omega)| = \frac{|\sigma^*|}{\overline{|\in^*|}} \quad (26)$$

where:

$\theta(\omega)$ = phase angle between applied stress and strain for frequency ω, degrees;

$\overline{\theta_\in}$ = average phase angle for all strain transducers, degrees;

θ_σ = stress phase angle, degrees;

$|E^*(\omega)|$ = dynamic modulus for frequency ω, kPa (psi);

$|\sigma^*|$ = stress magnitude, kPa (psi); and

$\overline{|\in^*|}$ = average strain magnitude.

13. DATA QUALITY

13.1. In addition to the dynamic modulus and phase angle, a product of the data analysis described in Section 12 has four data quality indicators: (1) standard error of the applied stress, (2) average standard error for the strain measurements, (3) uniformity coefficient for the strain measurements, and (4) uniformity coefficient for the phase angle measurements.

13.2. These data quality indicators can be used to assess the reliability of the data. Table 7 presents recommended limits for the data quality indicators.

Table 7—Recommended Limits for Data Quality Indicators

Indicator	Symbol	Equation	Limit
Standard error of the applied stress	$se(\sigma)$	9	≤10%
Average standard error of the measured strains	$se(\in)$	21	≤10%
Uniformity coefficient of the measured strains	$U_\in$	22	≤35%
Uniformity coefficient for the phase angle measurements	U_θ	23	≤3°

Note 8—Recommended limits based on research completed in NCHRP Project 9-29 may be revised in the future.

14. REPORT

14.1. *Report the following for each specimen at each combination of temperature and frequency tested*:

14.1.1. Test temperature,

14.1.2. Test frequency,

14.1.3. Dynamic modulus (from Equation 26),

14.1.4. Average phase angle between applied stress and measured strain (from Equation 25),

14.1.5. Average strain magnitude (from Equation 21),

14.1.6. Stress magnitude (from Equation 7),

14.1.7. Standard error of the applied stress (from Equation 10),

14.1.8. Average standard error of the measured strains (from Equation 22),

14.1.9. Uniformity coefficient for the strain measurements (from Equation 23), and

14.1.10. Uniformity coefficient for the phase angle measurements (from Equation 24).

APPENDIX—ASSESSMENT OF AIR VOID CONTENT UNIFORMITY

(Nonmandatory Information)

X1. PURPOSE

X1.1. This Appendix presents a procedure for assessing the uniformity of the air void content in test specimens produced for dynamic modulus testing.

X1.2. The approach tests the significance of the difference in mean bulk specific gravity between the top and bottom third of the specimen relative to the middle third.

X1.3. The procedure can be used to determine the height for preparing gyratory specimens with a specific compactor to minimize within sample variations in air voids.

X2. SUMMARY

X2.1. Three test specimens are prepared as described in Section 9 from gyratory specimens produced with the same mixture mass and compacted to the same height.

X2.2. The test specimens are cut into three slices of equal thickness, and the bulk specific gravity of each slice is determined.

X2.3. A statistical hypothesis test is conducted to determine the significance of differences in the mean bulk specific gravity of the top and bottom slices relative to the middle.

X3. PROCEDURE

X3.1. Prepare three test specimens following Section 9 to a target air void content of 5.5 percent. All three specimens shall have air void contents with the range of 5.0 to 6.0 percent.

X3.2. Label the top, middle, and bottom third of each specimen, then saw the specimens at the third points.

X3.3. Determine the bulk specific gravity of each of the nine test section slices in accordance with T 166 for dense- and gap-graded mixtures or T 269 for open-graded mixtures.

X3.4. Assemble a summary table of the bulk specific gravity data where each column contains data for a specific slice, and each row contains the data from a specific core.

X3.5. For each column, compute the mean and variance of the bulk specific gravity measurements using Equations X3.1 and X3.2.

$$\overline{y} = \frac{\sum_{i=1}^{3} y_i}{3} \quad (X3.1)$$

$$s^2 = \frac{\sum_{i=1}^{3} \left(y_i - \overline{y}\right)^2}{2} \quad (X3.2)$$

where:

$\overline{y}$ = slice mean;

y_i = measured values; and

s^2 = slice variance.

X3.6. *Statistical Comparison of Means*—Compare the mean bulk specific gravity of the top and bottom slices to the middle slice using the hypothesis tests described below. In the descriptions below, subscripts *t*, *m*, and *b* refer to the top, middle, and bottom slices, respectively.

X3.6.1. Check the top relative to the middle.

X3.6.2. *Null Hypothesis*—Mean bulk specific gravity of the top slice equals the mean bulk specific gravity of the middle slice:

$$\mu_t^2 = \mu_m^2$$

X3.6.3. *Alternative Hypothesis*—Mean bulk specific gravity of the top slice is not equal to the mean bulk specific gravity of the middle slice:

$$\mu_t^2 \neq \mu_m^2$$

X3.6.3.1. *Test Statistic*:

$$t = \frac{(\bar{y}_t - \bar{y}_m)}{0.8165(s)} \quad (X3.3)$$

where:

$$s = \sqrt{\frac{s_t^2 + s_m^2}{2}} \quad (X3.4)$$

$\bar{y}_t$ = computed mean for the top slices;

$\bar{y}_m$ = computed mean for the middle slices;

s_t^2 = computed variance for the top slices; and

s_m^2 = computed variance for the middle slices.

X3.6.3.2. *Region of Rejection*—For the sample sizes specified, the absolute value of the test statistic must be less than 2.78 to conclude that bulk specific gravity of the top and middle slices are equal.

X3.6.4. Check the bottom relative to the middle.

X3.6.4.1. *Null Hypothesis*—Mean bulk specific gravity of the bottom slice equals the mean bulk specific gravity of the middle slice:

$$\mu_b^2 = \mu_m^2$$

X3.6.4.2. *Alternative Hypothesis*—Mean bulk specific gravity of the bottom slice is not equal to the mean bulk specific gravity of the middle slice:

$$\mu_b^2 \neq \mu_m^2$$

X3.6.4.3. *Test Statistic*:

$$t = \frac{(\bar{y}_b - \bar{y}_m)}{0.8165(s)} \quad (X3.5)$$

where:

$$s = \sqrt{\frac{s_b^2 + s_m^2}{2}} \quad (X3.6)$$

$\bar{y}_b$ = computed mean for the bottom slices;

$\bar{y}_m$ = computed mean for the middle slices;

s_b^2 = computed variance for the bottom slices; and

s_m^2 = computed variance for the middle slices.

X3.6.4.4. *Region of Rejection*—For the sample sizes specified, the absolute value of the test statistic must be less than 2.78 to conclude that bulk specific gravity of the bottom and middle slices are equal.

X4. ANALYSIS

X4.1. Significant differences in the bulk specific gravity of the top and bottom slices relative to the middle indicate a systematic variation in density within the specimen.

X4.2. Specimens with differences of bulk specific gravity for the top or bottom slices relative to the middle slices on the order of 0.025 have performed satisfactorily in the dynamic modulus test.

X4.3. Changing the height of the gyratory specimen can improve the uniformity of the density in the test specimen.